Average income, income inequality and export unit values *

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Abstract

This paper analyses the relationship between a country's income distribution and its exports' unit values. Using bilateral export flows, we not only confirm the positive association between a country's average income and its export unit values, but further identify a heterogeneous relationship with income inequality: we find a greater income spread to be associated with higher exports unit values in the case of poor countries only. These results are robust to the inclusion of controls for other determinants of export unit values, as well as to the use of alternative measures of income inequality and of the quality index. We finally discuss various theoretical rationalizations for this heterogeneous relationship between income inequality and quality content of exports along the average income dimension, and show suggestive evidence that demand-side mechanisms can account for it at least partly.

Keywords: Income distribution, Export Unit Values, Product quality, Trade, Home market effect.

JEL classification: F12, L15, O15.

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1 Introduction

How the quality content of production and exports relates to the income distribution in the exporting country has been subject to debate over the past decades. Some studies emphasize that because of increasing skill intensity along the quality ladder in the production process, a country's specialisation in high-quality varieties results in higher earnings inequality. Following the Linder conjecture, other studies suggest that a country's income distribution determines both the relative demand for high- and low- quality varieties among its population, and its specialization along the quality ladder.

We here contribute to this debate by documenting and discussing the heterogeneous relationship between the quality content of exports and income inequality along average income in the exporting country. Using bilateral export flows at the 6-digit product level for more than 150 countries, we confirm that unit values increase significantly with the average income of the exporting country (controlling for importer-product-year fixed effects). We then highlight that controlling for average income, a greater income spread as measured by the Gini index is positively related to export unit values in poor countries only. This heterogeneous relationship between the quality content of exports and income inequality is robust to the use of various proxies for income per capita (i.e. human capital and TFP), income inequality (i.e. share of income accruing to the Top 10% and Top 20%) and quality (i.e. the quality index estimated by Feenstra and Romalis, 2014). It also resists the inclusion of various controls suggested by the literature (i.e. population and trade openness of the exporting country, as well as the bilateral distance between the trading partners). Based on a discussion of both supply- and demand-side theoretical explanations to these patterns, we finally show that controlling for earnings inequality, the heterogeneous relationship between the quality content of exports and income inequality remains remarkably stable. We take this last empirical result as suggestive evidence that demand-side explanations are relevant to understanding how income distribution interacts with vertical specialization across countries. The relationship between income inequality and export unit values is economically meaningful: a one standard-deviation of the Gini index in our data is associated to a -16.5% and a +12.2% variation of export unit values for the richest and the poorest country in our sample respectively. Providing a full-fledged quantification of the respective role of supply- and demand- side determinants in this relationship goes beyond the scope of this paper; but we believe our work provides an interesting step to go in this direction.

A wide body of literature has explored the determinants of the quality content of trade flows. Among contributions focusing on supply-side mechanisms, Schott (2004), Verhoogen (2008) and Fieler (2011a,b) have exemplified the impact of exporters' production technology and relative factor abundance on a country's export unit values, while Lugovskyy and Choi (2018) investigate the impact of credit constraints on export prices. When it comes to more demand-based explanations, Hallak (2006) shows that richer countries tend to import higher-price/higher-quality varieties from high income countries and Hallak (2010) shows that countries with similar income levels trade more together; he interprets both results in light of the Linder hypothesis. Combining both supply- and demand- side arguments, Hummels and Skiba (2004) as well as Lugovskyy and Skiba (2016) discuss the effect of distance between the trading partners on the quality content of trade: when part of the trade costs are additive, the deterring effect of distance on trade flows is less intense for high-quality varieties, a phenomenon known as the Alchian-Allen effect. Finally, Bekkers et al. (2012) as well as Flach and Janeba (2017) investigate the impact of a country's income inequality on its import unit values, while Choi et al. (2009) demonstrate that countries displaying similar income distributions tend to exhibit similar distributions of import prices.

As it is clear from the papers listed above, some studies focus on the exporter side, others on the importer side, and some on both. In this paper, we investigate the exporter side, and are the first to exploit variations across a wide range of low-, middle- and high-income countries to identify a heterogenous relationship between income inequality and export quality along the average income dimension. Within the theoretical literature, two strands of models could explain the relationship between export quality on the one hand, and average income and income inequality on the other hand. On the supply-side, models where high-income countries are more skill-abundant and/or models *a la* Verhoogen (2008) where the production of high-quality varieties is relatively skill-intensive both predict that countries that produce high-quality varieties are richer and more unequal. On the demand side, Fajgelbaum et al. (2011) exemplify the existence of a "**vertical** home market effect" leading to an impact of a country's average income and income inequality levels on the quality of its exports. The essence of the mechanism is simple: similarly to what has been identified by the economic geography literature in the case of trade in horizontally differentiated varieties, production is expected to follow demand in presence of economies of scale and positive trade costs. The only difference is that when preferences are non-homothetic, income distribution (i.e. average income and income inequality), and not only total income, is affecting the relative size of domestic demand for high- and low-quality varieties, and thus influences the specialisation of countries in terms of quality. Empirically, Dingel (2017) adopts a structural approach to disentangle and quantify the two potential mechanisms (supply-based relative factor abundance vs. demand-based home-market effect), but he does not discuss the heterogenous relationship between inequality and export quality we identify in our study. Our paper instead discusses how supply- and demand-based mechanisms can account for the non-linear relationship between export quality and income inequality we uncover empirically; and we adopt a reduced-form approach to isolate more specifically the presence of demand-side mechanisms by combining information on both income and earnings inequality.

The rest of the paper is structured as follows. Our data and empirical results are presented in sections 2 and 3. We then discuss the theoretical frameworks able to rationalize these facts in section 4 while section 5 concludes.

2 Data and empirical strategy

We present in this section the data we use and the empirical strategy we follow to describe the relationship between a country's income distribution and the unit value of the goods it exports.

2.1 Data

To conduct our empirical analysis, we need information on countries's exports at a detailed level of the product nomenclature, on their average income and income inequality, and on various other country-level characteristics.

For information on trade flows, we use the Comtrade data for the years 2006, 2008, 2010 and 2012.[1] The data records all bilateral trade flows at the 6-digit level of the HS nomenclature, in value (dollars) and in volume (tons). As commonly done in the literature, we clean the data and drop those flows for which the declared value is below 10,000 dollars or whose information on value or quantity is 0 or missing. Moreover, it is well known that information on unit values can be particularly noisy. For a given product and a given year, we thus drop all the flows whose unit value is below the 5th percentile or above the 95th percentile observed across all the flows in the data for that product and that year. Regarding income distribution in the exporting country, we use the data from the World Development Indicators of the World Bank. Population and total GDP allow us to compute the GDP per capita of each country. We complement this information with three measures of income inequality: the Gini index and the share of income accruing to the top 10% and the top 20% of the population in terms of income. Other characteristics of the exporting countries are taken from the Penn World Table, in particular country-level TFP and average human capital of the population (measured by an index based on years of schooling and returns to education). Finally, information on distance between countries is taken from a CEPII database[2] and an alternative measure of product quality described later in the paper is taken from a database built by Feenstra and Romalis (2014).[3]

Note that the data on income inequality notoriously suffers from many missing observations. To limit this issue, we smooth all the country-level explanatory variables by computing moving averages across three years (the current year and the years before and after).[4] In the end, 151 exporting countries are present in at least one of the regressions presented in this paper.[5]

[1] We do not keep all the years because of the very high number of observations. Since all our regressions exploit cross-sectional variations in the data, this is not an issue for the estimation.

[2] "dist_cepii" database available online at: `http://www.cepii.fr/francgraph/bdd/distances.html`.

[3] Available at: `https://cid.econ.ucdavis.edu/Html/Quality_Data_Page.html`.

[4] For example, in our data, the Gini index is available for 74 countries only in 2006. If we take the average Gini index for the year 2005, 2006 and 2007, the information becomes available for 106 countries.

[5] See Table 1 in Appendix B for the entire list.

2.2 Estimated equation

We want to relate the unit value of exports to the income distribution in the *exporting* country. We run the analysis at the exporter-importer-product-year level and estimate the following equation:

$$\mathrm{uv}_{xmpt} = \alpha \mathrm{X}_{xt} + \beta \mathrm{Z}_{xmt} + \mu_{mpt} + \epsilon_{xmpt}$$

where uv_{xmpt} is the log unit value of exports of product p by country x to country m at time t, X_{xt} is a vector of exporter-year characteristics, Z_{xmt} is a set of bilateral determinants of unit values, μ_{mpt} is an importer-product-year fixed effect and ϵ_{xmpt} is the error term. Given the presence of the fixed effect μ_{mpt}, the effect of the other variables is estimated by comparing, for a given importer and a given product in a given year, the unit values of the varieties coming from different source countries.

The vector X_{xt} contains the following variables: log income per capita, a measure of income inequality, log population, and a measure of trade openness in the exporting country.

In line with previous evidence, we expect the log income per capita to be positively related to unit values since richer countries have been shown to have a comparative advantage in high-quality and thus high-price varieties (due to both supply- and demand-side mechanisms, as emphasized, e.g., in Schott, 2004; Dingel, 2017). We will also check that the patterns we uncover hold when using the exporter's human capital and TFP indexes from the Penn World Tables instead of income per capita. The prior is less clear for income inequality. From a supply-side perspective, it has been shown that high-quality varieties are skill-intensive so that the production of such varieties is associated with higher wage inequality (see, e.g., Verhoogen, 2008). On the other hand, from a demand-side perspective, Fajgelbaum et al. (2011) show that the effect of income inequality on the specialization of countries along the quality-ladder is ambiguous. This is because all else equal, higher inequality means more wealthy, but also more poor consumers; only under certain conditions does this unambiguously translate into higher demand for (and greater specialization in) high-quality varieties. The correlation between population size and the average quality-level of production is also *a priori* unclear. Fajgelbaum et al. (2011) show that an increase in population increases disproportionately the number of varieties that are more horizontally differentiated. Since we can reasonably think that high-quality varieties are more differentiated than low-quality ones, this argument points at a possible positive correlation between export unit values and population size. On the other hand, Desmet and Parente (2010) show that bigger markets exhibit lower markups and consequently bigger firms, which favors process innovation. This could lead, all else equal, to lower prices in bigger countries. Given these conflicting theoretical insights, we have no prior on the empirical correlation between unit values on the one hand, and income inequality and population on the other hand. Finally, we introduce the ratio of total exports plus imports over GDP as a measure of trade openness. This control has two main purposes. First, trade openness might directly affect the quality content of exports. Indeed, trade models with firm heterogeneity in terms of quality highlight a selection mechanism where high-productivity/high-quality firms are the most likely to export. In such models, the quality produced by the marginal exporter decreases as trade becomes easier (Crozet et al., 2012; Johnson, 2012; Feenstra and Romalis, 2014); this selection mechanism then generates a negative correlation between trade openness and the average quality of the export basket. Second, trade openness might not only affect the quality content of exports but also earnings and earnings inequality both across and within firms. This effect channels through: i) an increase in the wages paid by internationally active (and thus more profitable) firms as compared to domestic ones (Amiti and Davis, 2012; Helpman et al., 2017), ii) a skill-biased technology and quality upgrading, the firms in low- and middle- income countries needing to adapt their processes and the quality content of their production so as to serve the richer countries when they open to trade (Bustos, 2011; Bas, 2012; Iacovone and Javorcik, 2012). Since we are interested in the correlation between income distribution and the quality of exports beyond the direct effect of trade on both dimensions, we prefer controlling directly for trade openness in our regressions.

The set of bilateral determinants of unit values Z_{xmt} only contains the bilateral distance between the exporting and the importing country. It is now well established that unit values of exports (both at the country-product and firm-product level) are positively related to the bilateral distance between the trading partners (Hummels and Skiba, 2004; Baldwin and Harrigan, 2011; Bastos and Silva, 2010; Martin, 2012; Manova and Zhang, 2012). This feature is in line with the so called Alchian-Allen effect where high quality varieties tend to be relatively more exported to distant countries. This effect might arise due to the selection of high-quality firms in more difficult destinations (Johnson, 2012), or to a demand-side mechanism when part of the trade costs are additive and thus relatively less important for more expensive varieties (for a thorough discussion of the Alchian-Allen effect and its relationship with export unit values, see Lugovskyy and Skiba, 2016). We thus expect the coefficient on distance to be positive in our regressions.

We present in Table 2 in Appendix C a correlation table for all the variables we take into account in our empirical analysis. The variable of interest in these regressions will be the proxy for income inequality. We will show that our results are robust to the use of three alternative measures of income inequality: the Gini index and the share of income in the hands of the top 10% and top 20% of the population in the income distribution.

Finally, our dependent variable is exporter-importer-product-year specific, while our variables of interest are exporter-year specific. According to Moulton (1990), standard-errors of the coefficients on exporter-year characteristics might consequently be downward-biased. To correct for this, we cluster all the regressions at the exporter-year level.

3 Results

In this section, we first present some descriptive statistics on average income and income inequality in the exporting countries present in our sample. We then detail our baseline results and provide several additional robustness checks.

3.1 Descriptive statistics

Table 1 shows there is a great heterogeneity in our sample both in terms of average income and income inequality. While average income is equal to nearly 14,000$, the median is much lower at around 4,900$ over the period under study. This shows that the distribution of average income across countries is very much right skewed, which is reflected in a relatively high standard deviation as compared to the mean (and thus a high coefficient of variation). Based on the categories established by the World Bank in 2006 (first year in our sample), there are 47 low- , 77 middle- and 27 high- income countries in our sample. Regarding income inequality, the heterogeneity is also high even though less massive than the one observed for average income (the coefficient of variation is respectively equal to 22.8%, 22.5% and 16.3% for the Gini index and the share of income accruing to the top 10% and the top 20%).

Table 1: Average income and income inequality in the estimation sample

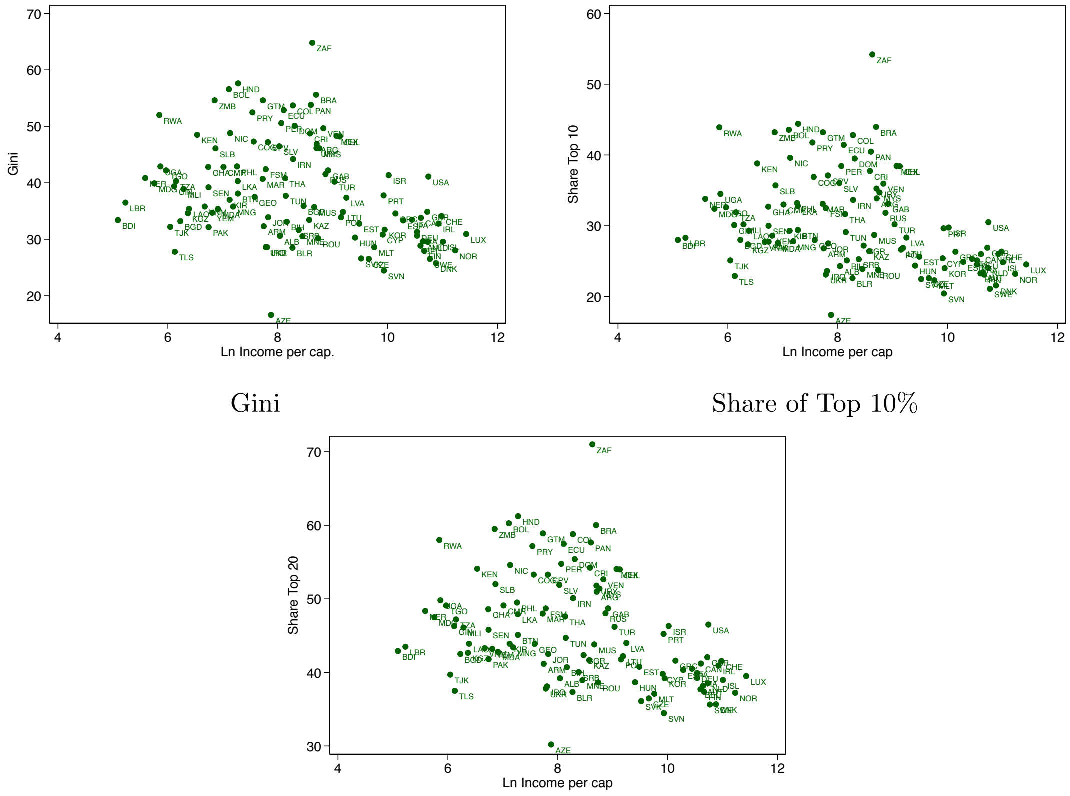
	Mean	Med	Min	Max	Sd
Average income	13993.65	4871.73	148.36	109169.8	19808.96
Gini	38.11	35.8	16.4	64.8	8.68
Share of top 10%	29.98	28.00	17.25	54.2	6.74
Share of top 20%	45.23	43.37	15.97	71	7.35

Figure 1 plots our three measures of income inequality against the log of average income. The picture is very similar for the three measures:[6] there is a slight negative correlation between average income and income inequality, but overall, for any level of income inequality, we observe countries with very different levels of average income. We thus clearly have enough variation in the data to disentangle the correlation between export unit values and these two dimensions of the income distribution in the exporting country.

3.2 Export unit values and income distribution within the exporting country

Table 2 displays our baseline results. All regressions include importer-product-year fixed effects. In column (1), we only include the income distribution in the exporting country on the top of the fixed effects; income per capita is positively and significantly correlated with export unit values, while inequality is on the opposite negatively correlated

[6] Which is not surprising since the pairwise correlation between any two of the three measures is above 97% (unreported computations available upon request).



Gini

Share of Top 10%

Share of Top 20%

Figure 1: Average income and income inequality in 2006

with export prices. We include additional exporter characteristics in column (2). Bigger countries tend to export cheaper varieties, as suggested by Desmet and Parente (2010). More expensive varieties tend to be shipped to more distant countries, which is coherent with the extensive literature on the Alchian-Allen effect we already cited. Finally, our measure of trade openness is slightly negatively correlated with export unit values, which lends support to the idea that greater openness to trade allows low price/low quality producers to enter the export markets (Crozet et al., 2012; Johnson, 2012; Feenstra and Romalis, 2014). Controlling for all of these covariates, average income is still positively and very robustly associated with export unit values, and the coefficient on the Gini index remains negative and significant.

We then test for possible heterogeneity in the relationship between export unit values and income inequality along average income. We thus introduce an interaction term between the Gini index and income per capita. Results in column (3) show that the coefficient on the Gini index is now positive while its interaction with income is negative and significant: income inequality is positively associated with export unit values in poor countries only, which is further emphasized in column (4), where we interact the Gini index with dummies identifying middle- and high- income countries.[7]

We finally replicate the analysis replacing income per capita by the exporter-level human capital and TFP indexes taken from the Penn World Tables. The results presented in columns (5) and (6) of Table 2 are remarkably stable and the main message holds:

[7]Poor countries are thus the excluded category, so that the coefficient on the Gini index alone corresponds to the coefficient for these low-income countries.

Table 2: Bilateral export unit values and exporter characteristics

	Ln uv_{xmpt}					
	(1)	(2)	(3)	(4)	(5)	(6)
GDP per cap. (Ln)	0.168^a	0.155^a	0.327^a	0.238^a		
	(0.016)	(0.015)	(0.063)	(0.024)		
Human capital index					0.554^a	
					(0.141)	
TFP						1.348^a
						(0.369)
Gini	-0.006^a	-0.006^a	0.039^b	0.011^a	0.015	0.006
	(0.002)	(0.002)	(0.017)	(0.003)	(0.011)	(0.008)
Gini × GDP per cap. (Ln)			-0.005^a			
			(0.002)			
Gini × Human capital index					-0.008^b	
					(0.004)	
Gini × TFP						-0.025^b
						(0.012)
Gini × Middle inc. country				-0.013^a		
				(0.002)		
Gini × High inc. country				-0.015^a		
				(0.003)		
Population (Ln)		-0.038^a	-0.033^b	-0.042^a	-0.062^a	-0.063^a
		(0.014)	(0.015)	(0.013)	(0.014)	(0.014)
Bilateral distance (Ln)		0.062^a	0.063^a	0.062^a	0.066^a	0.073^a
		(0.005)	(0.005)	(0.005)	(0.006)	(0.006)
Trade openness		-0.028^c	-0.039^b	-0.023	-0.093^a	-0.081^a
		(0.015)	(0.015)	(0.016)	(0.018)	(0.016)
Importer-Product (HS 6-digit)-Year fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Observations	10,277,544	10,277,544	10,277,544	10,277,544	10,197,044	10,018,400
R-squared	0.862	0.863	0.864	0.864	0.862	0.861

Standard errors clustered at the exporter-year level.
^{a} p<0.01, ^{b} p<0.05, ^{c} p<0.1

richer/more educated/more productive countries export more expensive varieties, while income inequality is positively related to unit values, but in poor/less educated/less productive countries only.

3.3 Robustness checks

In this section, we show that the heterogeneous relationship between the quality content of exports and income inequality along average income is robust to several checks.

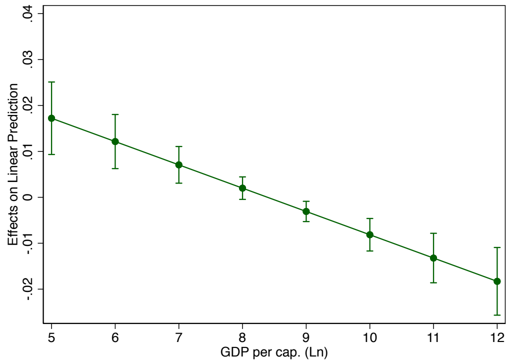
First, our results are not sensitive to the income inequality measure we use. We propose two alternative proxies: the share of income accruing to the top 10% and to the top 20% of the population. For the sake of brevity, we only reproduce in Table 3 the regression with the interaction term between average income and inequality. Whatever the measure of income inequality, the non-linear relationship is there. The estimated level of income above which income inequality stops being positively related to export unit values is comprised, in log, between 8 and 9.5, i.e. between 2,400 and 14,800$ approximately (within the range of average income observed for middle-income countries according the classification of the World Bank).[8]

Table 3: Bilateral export unit values and exporter characteristics - Various measures of income inequality

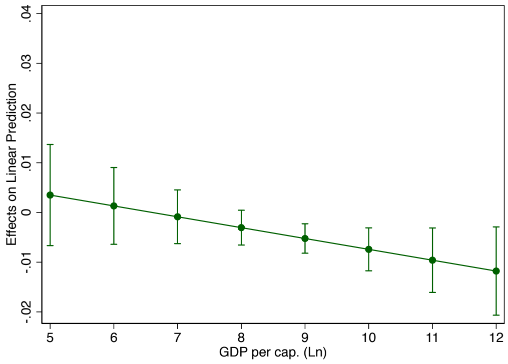
	Ln uv$_{xmpt}$		
	(1)	(2)	(3)
GDP per cap. (Ln)	0.327^a	0.405^a	0.610^a
	(0.063)	(0.065)	(0.103)
Gini	0.039^b		
	(0.017)		
Gini × GDP per cap. (Ln)	-0.005^a		
	(0.002)		
Share of inc. owned by top 10		0.078^a	
		(0.022)	
Share of inc. owned by top 10 × GDP per cap. (Ln)		-0.009^a	
		(0.003)	
Share of inc. owned by top 20			0.096^a
			(0.023)
Share of inc. owned by top 20 × GDP per cap. (Ln)			-0.010^a
			(0.002)
Population (Ln)	-0.033^b	-0.034^b	-0.028^b
	(0.015)	(0.015)	(0.012)
Bilateral distance (Ln)	0.063^a	0.060^a	0.060^a
	(0.005)	(0.005)	(0.005)
Trade openness	-0.039^b	-0.027	-0.021
	(0.015)	(0.017)	(0.017)
Importer-Product (HS 6-digit)-Year fixed effects	Yes	Yes	Yes
Observations	10,277,544	10,277,544	10,277,544
R-squared	0.864	0.863	0.864

Standard errors clustered at the exporter-year level.
^{a} p<0.01, ^{b} p<0.05, ^{c} p<0.1

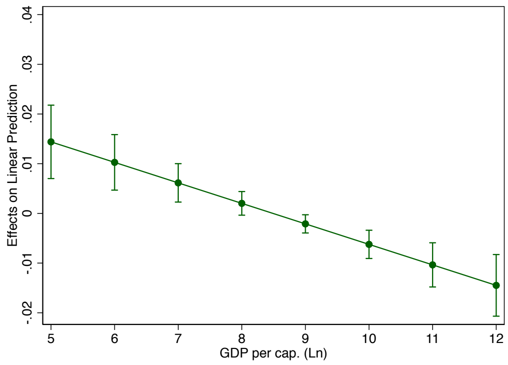
[8] Given the specification, this income threshold is equal (in log) to $\frac{\widehat{\text{coef. inc. ineq.}}}{\widehat{\text{coef. inc. ineq.}} \times \text{avg inc.}}$. Based on our estimations, the smallest value is obtained for the Gini index, and the biggest one for the share of income accruing to the top 20% of the population.



All products



Highly elastic products



Homogeneous products

Figure 2: Marginal effect of the Gini index

If export unit values are a proxy for quality, we can expect the relationship between income distribution and unit values to be less intense for products that are less differentiated. We use the elasticities of substitution estimated by Imbs and Mejean (2015) as a measure of product differentiation. We define as non-differentiated products those whose elasticity of substitution is above 6.5.[9] We estimate the regression in column (3) of Table 2 for these highly substitutable products. We do the same for the products defined as (strictly) homogeneous in the Rauch classification. Based on the coefficients we obtain, we can compute the marginal effect of the Gini index at each income level.[10] Figure 2 presents the estimated marginal effect of the Gini index for all the products, highly elastic ones and (Rauch-based) homogeneous ones. At each income level, the marginal effect of income inequality is lower in absolute value for highly elastic and homogeneous products than for the entire sample of products; moreover, the slope of the marginal effect is lower in absolute value for these less differentiated products. We provide in Appendix B the same graphs computed with the share of income owned by the top 10% and the top 20% respectively: the picture we obtain is very much the same. This shows that income distribution relates to export unit values for the products that are more vertically differentiated, corroborating the interpretation in terms of quality.

Still, we know that the correlation between unit values and quality is not perfect, other factors such as the production costs or the exchange rate determining export prices. This

[9] 6.5 corresponds to the 9th decile in the distribution of the elasticities estimated by Imbs and Mejean (2015).

[10] Note that for computational reasons, we run these regressions at the exporter-product-year level, instead of the bilateral level.

is why several procedures have been developed to recover quality measures from trade data (Khandelwal, 2010; Hallak and Schott, 2011; Khandelwal et al., 2013). Khandelwal (2010) in particular shows that the shorter the product quality ladder, the lower the correlation between unit values and his quality index. The derivation of these indexes mostly relies on the demand side, following the intuition that conditional on price, varieties with a higher market share must be higher-quality varieties. Feenstra and Romalis (2014) propose a framework with a richer supply side (where the so-called Alchian-Allen effect is taken into account as well as the selection of high-productivity/high-quality firms into exporting) and where the demand side allows for non-homothetic preferences. They perform the analysis at the SITC 4-digit level. Their estimated country-sector quality indices are available online. We use them for the period 2005-2011 as a dependent variable and reproduce the analysis of section 3.2.

For the purpose of brevity, we directly reproduce the specification with all the controls and the interaction term between average income and inequality. As can be seen in Table 4, all the results go through: income per capita in the exporting country is positively related to the quality of exports, while the correlation with income inequality is highly non linear, positive and significant in poor countries only.

Among the controls, bigger countries still appear to export lower-quality goods on average. Quite interestingly, it also seems that the countries that are more open to trade tend to export lower quality varieties, again well in line with the prediction in Crozet et al. (2012), Johnson (2012) and Feenstra and Romalis (2014): when trade becomes easier, more firms enter the markets, and the marginal exporter tends to export lower quality varieties.[11] Finally, we can note that given the coefficients we obtain on income inequality and its interaction with the log GDP per capita, the average income above which quality and income inequality are not positively related anymore is estimated to lie between 4,000 and 13,400$, i.e. again in the range of average income values observed in middle-income countries.

3.4 Quantitative assessment

To get a sense of how much average income and inequality contribute to the variations in export unit values observed across countries, we use the regression coefficients presented in column (3) of Table 2 and the information contained in Table 1 to compute the percentage change in unit values associated with a one standard deviation increase in both variables. We find that for the average country in our sample in terms of average income and Gini index, a one standard deviation increase in average income is associated with a 12.0% increase in export unit values, while a one standard deviation increase in the Gini coefficient is associated with a 7.6% decrease in unit values. However, the contribution of income inequality varies a lot across countries, from -16.5% for the richest country to +12.2% for the poorest country in our sample.

[11] Bilateral distance does not appear here among the regressors since the Feenstra-Romalis index is exporter-industry specific, and not bilateral.

Table 4: Quality of exports and exporter characteristics

	Feenstra-Romalis quality index$_{xpt}$		
	(1)	(2)	(3)
GDP per cap. (Ln)	0.255^a	0.297^a	0.385^a
	(0.035)	(0.034)	(0.051)
Gini	0.038^a		
	(0.009)		
Gini × GDP per cap. (Ln)	-0.004^a		
	(0.001)		
Share of inc. owned by top 10		0.061^a	
		(0.011)	
Share of inc. owned by top 10 × GDP per cap. (Ln)		-0.007^a	
		(0.001)	
Share of inc. owned by top 20			0.058^a
			(0.011)
Share of inc. owned by top 20 × GDP per cap. (Ln)			-0.007^a
			(0.001)
Population (Ln)	-0.066^a	-0.066^a	-0.065^a
	(0.007)	(0.007)	(0.007)
Trade openness	-0.283^a	-0.276^a	-0.275^a
	(0.028)	(0.028)	(0.028)
Product (SITC 4-digit)-Year fixed effects	Yes	Yes	Yes
Observations	366,225	366,225	366,225
R-squared	0.175	0.175	0.175

Standard errors clustered at the exporter-year level.
^{a} p<0.01, ^{b} p<0.05, ^{c} p<0.1

Average income and income inequality are thus both statistically and economically significantly related to export unit values, the sign of the relationship between unit values and income inequality being different for rich and poor countries.

4 Supply vs. demand-side mechanisms: a theoretical discussion of our results

All of the results presented above highlight a robust non-linear relationship between unit values/quality content of exports and income inequality in the exporting country. Controlling for average income, income inequality is positively related to the quality content of exports in poor countries only. In this section, we provide a discussion of the theoretical mechanisms that might drive our results. More precisely, we first identify two potential families of models that could account for such an empirical regularity: supply-based models where the specialisation in skill-intensive goods results in higher wage inequality, and demand-based models where the distribution of income drives a country's export specialisation. We then provide further empirical evidence that shows, we believe, the presence of demand-based mechanisms in the determination of a country's quality of exports.

4.1 Two main potential theoretical mechanisms

From a theoretical perspective, two types of explanations can help rationalize the patterns we highlight.

One is based on supply-side mechanisms. If, as assumed by Verhoogen (2008) for example, the production of high-quality varieties is relatively intensive in skilled labor, a country that specializes in high-quality varieties should exhibit, all else equal, higher wage inequality. However, to be consistent with the non-linear relationship between quality and inequality along average income, this should be true for poor countries only. In this family of models, causality flows from export specialisation to income distribution, and more specifically, to wage distribution.

The other explanation is based on demand-side mechanisms, and has its roots in the "**vertical** home market effect" literature (Linder, 1961). In the presence of economies of scale and positive trade costs, production is expected to follow demand: when preferences are non-homothetic, income distribution then affects the vertical specialisation of countries through its impact on the relative size of domestic demand for high- and low-quality varieties (Fajgelbaum et al., 2011). While we believe the contribution of our paper to be mainly empirical, we anyway provide in Appendix C a model of intra-industrial trade in vertically differentiated varieties in which income distribution determines the quality of a country's export basket. In such a framework, it is possible to show that the quality content of exports increases with the level of income inequality in the exporting country *only* in the case of "poor" countries, i.e. with a low enough average income level. Such results are highly reminiscent of those obtained by Fajgelbaum et al. (2011); the two frameworks however differ in the way the non-homotheticity of consumers' preferences (which is necessary to obtain an impact of income distribution on the composition of aggregate demand) is modelled: our model features love-for-variety, while Fajgelbaum et al. (2011) assume variety of taste.[12]

The reason why we provide an alternative framework is that in our model, the non-linear relationship between the quality content of production and income inequality can be very intuitively related to the shape of the Engel curve representing the share of a consumer's expenditures devoted to high-quality varieties (Figure 3). Consider a world composed of two types of consumers, rich an poor. For a given average income, an increase in inequality means both more rich and more poor consumers; the net effect on the overall demand for quality is a priori undetermined, and depends on how the demand for high-quality varieties evolves along income for both income groups. As demonstrated in Appendix C and illustrated in Figure 3, the quality content of production and exports actually increases unambiguously with income inequality *only* when the share of income devoted to high-quality goods is increasing and convex along income for both rich and poor consumers: this is because only in this case the additional expenses of rich consumers on high-quality

[12]Such a result equivalence between a framework featuring heterogeneous consumers and unit consumption and models with love-for-variety at the individual level is reminiscent of the the one demonstrated by Anderson et al. (1992) in a horizontal framework.

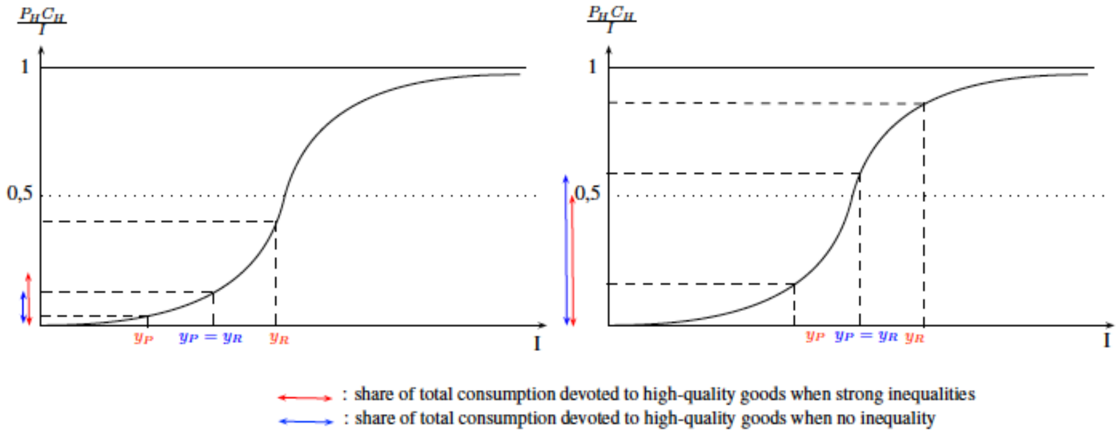


Figure 3: Heterogenous impact of inequality along the average income dimension

varieties clearly more than compensate the lower expenses of poor consumers. And we can show that this convexity of the Engel curve for high-quality varieties is more likely when the average income level is low enough, grounding theoretically the patterns we uncover in the data. In this second family of models, causality flows from income distribution to export specialisation.

Ideally, in order to disentangle the supply- and demand-based mechanisms that drive our results and to push further the causal inference on the relationship between income distribution and the quality content of exports, one would like to exploit exogenous variations of average income and income inequality; however, variables that effect average income and income inequality without affecting directly the quality of production are not easy to find. Another potential avenue would be to provide a more structural approach as in Dingel (2017), but we would need micro-data on firm-level production, shipments and factor-use for multiple countries; this is beyond the scope of this paper. We rather present in the next subsection a reduced-form approach aimed at isolating the potential effect of the composition of demand on export quality.

4.2 Disentangling supply- and demand-based mechanisms: distinguishing between wage and income inequality

One way to disentangle the supply- and demand- side mechanisms that connect the quality content of exports with inequality would be to distinguish wage inequality from income inequality in the data. Indeed, the theoretical and empirical literature that emphasized the effect of globalization on quality upgrading and inequality considers *earnings* inequality. However, in our data, income inequality measures are based on the total disposable income of households, calculated by adding together the personal income received by all the household members plus the income received at the household level. It encompasses earnings from work, but also private income from investment and property, transfers between households and all social transfers received in cash including old-age and pensions.

Therefore, our measures of income and income inequality go beyond wage and wage inequality. Actually, based on data from the International Labor Organization for 95 developed and developing economies, we find that the median ratio of the white collar to the blue collar earnings is equal to 2.04.[13] As a comparison, the median ratio of the average income of people in the top 10% to the average income of the rest of the population is much higher in our sample, equal to 3.5 (3.1 for the ratio of the top 20%). The figures are even more striking if we consider the income ratio of the top to bottom 10% and 20%, whose median value is respectively equal to 10.7 and 3.7. This is not surprising since as emphasized by Atkinson et al. (2011), "aggregate economic growth per capita and Gini inequality indexes are sensitive to excluding or including top incomes". They show that top incomes play a key role in the evolution of inequality in the past decades, the evolution of top incomes themselves being mainly driven by top managers' and CEOs' wages in some countries, and by capital income in other countries (in Scandinavia in particular). In addition, Philippon and Reshef (2013) point at the role of the financial sector in the evolution of wages and inequality. The earnings inequality induced by quality upgrading in the manufacturing sector is thus quite distinct from the income inequality in the population as a whole.

Table 5: Income *vs* earnings inequality

	Ln uv_{xmpt}			Feenstra-Romalis quality $index_{xpt}$		
	(1)	(2)	(3)	(4)	(5)	(6)
GDP per cap. (Ln)	0.447^a	0.450^a	0.452^a	0.438^a	0.431^a	0.430^a
	(0.090)	(0.089)	(0.087)	(0.061)	(0.060)	(0.062)
Gini	0.072^a	0.073^a	0.072^b	0.089^a	0.087^a	0.088^a
	(0.025)	(0.025)	(0.029)	(0.015)	(0.015)	(0.017)
Gini × GDP per cap. (Ln)	-0.008^a	-0.009^a	-0.008^a	-0.010^a	-0.009^a	-0.010^a
	(0.003)	(0.003)	(0.003)	(0.002)	(0.002)	(0.002)
Wage ratio		0.006	0.038		-0.040	-0.060
		(0.029)	(0.290)		(0.028)	(0.215)
Wage ratio × GDP per cap. (Ln)			-0.004			0.002
			(0.034)			(0.026)
Population (Ln)	0.000	0.000	0.000	-0.072^a	-0.074^a	-0.074^a
	(0.012)	(0.012)	(0.012)	(0.010)	(0.010)	(0.010)
Bilateral distance (Ln)	0.080^a	0.080^a	0.080^a			
	(0.005)	(0.005)	(0.005)			
Trade openness	-0.047^a	-0.049^b	-0.048^b	-0.312^a	-0.285^a	-0.286^a
	(0.016)	(0.019)	(0.019)	(0.034)	(0.038)	(0.039)
Importer-Product (HS 6-digit)-Year fixed effects	Yes	Yes	Yes	n.a.	n.a.	n.a.
Product (SITC 4-digit)-Year fixed effects	Yn.a.	n.a.	n.a.	Yes	Yes	Yes
Observations	2,870,193	2,870,193	2,870,193	99,560	99,560	99,560
R-squared	0.874	0.874	0.874	0.383	0.384	0.384

Standard errors clustered at the exporter-year level.
^{a} p<0.01, ^{b} p<0.05, ^{c} p<0.1

We use the information from the ILO on the skilled to unskilled workers wage ratio between 2006 and 2012 (available for 52 countries out of 151 in our sample) and introduce it

[13] We consider as white collars the managers, the professionals and the technicians and associate professionals. We define as blue collars the craft and related trades workers, the plant and machine operators and assemblers, and the elementary occupations.

as a additional control in our benchmark regressions. The correlation between this measure of wage inequality and our proxies for income inequality is equal to 0.33 at most. There is thus space to disentangle empirically the relationship between unit values/quality content of exports and income inequality from the one with earnings inequality. If one agrees that the potential effect of quality upgrading on wage inequality is captured by this control, any remaining statistical relationship between export unit values/Feenstra-Romalis index and income inequality should most likely be explained by demand-side mechanisms. The results presented in Table 5 show that whatever the dependent variable is (export unit values or Feenstra-Romalis quality index): i) the benchmark results on the non-linear relationship between the quality content of exports and income inequality of the exporting country still hold on the subsample of countries for which we have information on earnings inequality; ii) these results are barely affected by the inclusion of earnings inequality and its interaction with GDP per capita. The fact that income inequality remains positively related to the quality content of exports in poor countries only when wage inequality is controlled for suggests that demand-side explanation is a good candidate to rationalize, at least partly, the patterns we uncover in the data.

5 Conclusion

In this paper, we have shown that income distribution in a given country is significantly related to the quality content of its exports. If average income is non-ambiguously and strongly related to export unit values, the message is more subtle for income inequality: controlling for income, income inequality is associated with a higher quality content of exports for poor countries only. This non-linear pattern is robust to several controls and robustness checks. While both supply- and demand- side mechanisms could explain such patterns, suggestive evidence shows that demand-side explanations partly drive the results.

We believe we are the first to highlight empirically this heterogeneous relationship between the quality content of exports and income inequality. Given the growing academic and policy interest for the determinants and the consequences of income inequality, we think this is a valuable contribution that could be extended in several dimensions. In particular, the effect of income distribution on the composition of the aggregate demand might not only matter for the quality content of exports, but also for other outcomes such as the demand for (and thus public expenses on) healthcare and education for example.

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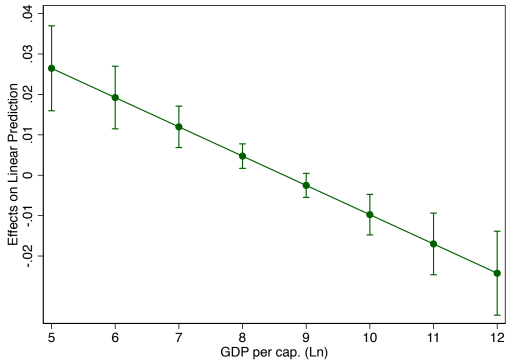
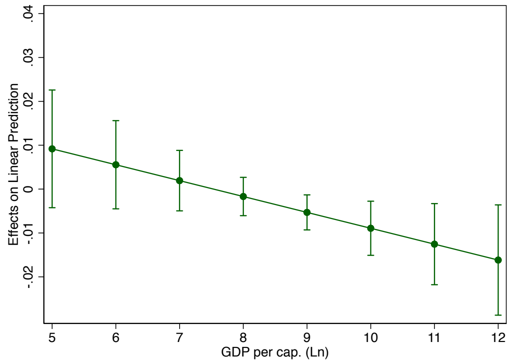
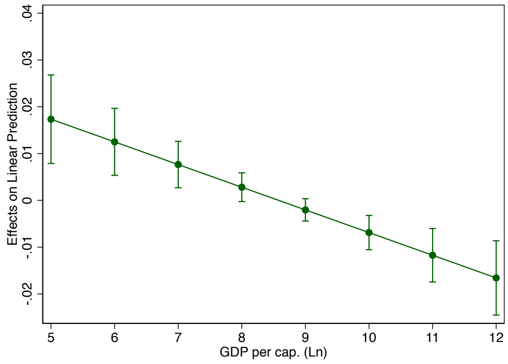


All products



Highly elastic products

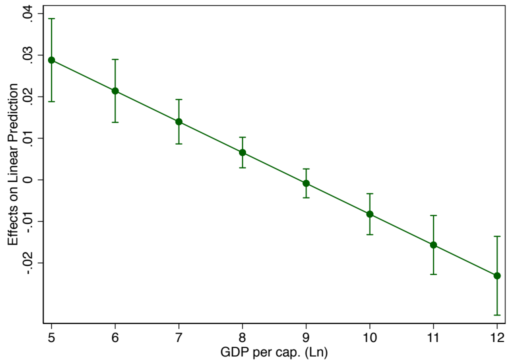


Homogeneous products

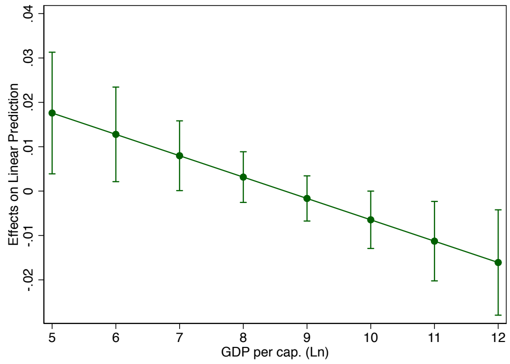
Figure 1: Marginal effect of the Top 10% share of income

APPENDIX

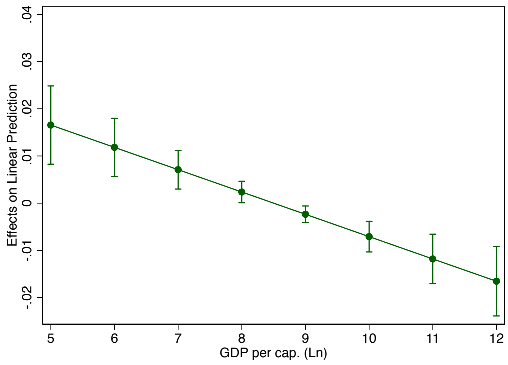
Appendix A: Marginal effect of income inequality and average income



All products



Highly elastic products



Homogeneous products

Figure 2: Marginal effect of theTop 20% share of income

Appendix B: Additional tables

Table 1: List of exporting countries in the estimation sample

ISO3 code	Country	ISO3 code	Country
AGO	Angola	ALB	Albania
ARG	Argentina	ARM	Armenia
AUS	Australia	AUT	Austria
AZE	Azerbaijan	BDI	Burundi
BEL	Belgium	BEN	Benin
BFA	Burkina Faso	BGD	Bangladesh
BGR	Bulgaria	BIH	Bosnia Herzegovina
BLR	Belarus	BOL	Bolivia
BRA	Brazil	BTN	Bhutan
BWA	Botswana	CAF	Central African Rep.
CAN	Canada	CHE	Switzerland
CHL	Chile	CHN	China
CIV	Côte d'Ivoire	CMR	Cameroon
COD	Dem. Rep. of the Congo	COG	Congo
COL	Colombia	COM	Comoros
CPV	Cape Verde	CRI	Costa Rica
CYP	Cyprus	CZE	Czech Rep.
DEU	Germany	DJI	Djibouti
DNK	Denmark	DOM	Dominican Rep.
DZA	Algeria	ECU	Ecuador
EGY	Egypt	ESP	Spain
EST	Estonia	ETH	Ethiopia
FIN	Finland	FJI	Fiji
FRA	France	FSM	Micronesia
GAB	Gabon	GBR	United Kingdom
GEO	Georgia	GHA	Ghana
GIN	Guinea	GMB	Gambia
GNB	Guinea-Bissau	GRC	Greece
GTM	Guatemala	HND	Honduras
HRV	Croatia	HTI	Haiti
HUN	Hungary	IND	India
IRL	Ireland	IRN	Iran
IRQ	Iraq	ISL	Iceland
ISR	Israel	ITA	Italy
JAM	Jamaica	JOR	Jordan
JPN	Japan	KAZ	Kazakhstan
KEN	Kenya	KGZ	Kyrgyzstan
KIR	Kiribati	KOR	Rep. of Korea
LAO	Lao People's Dem. Rep.	LBN	Lebanon
LBR	Liberia	LKA	Sri Lanka
LSO	Lesotho	LTU	Lithuania
LUX	Luxembourg	LVA	Latvia
MAR	Morocco	MDA	Rep. of Moldova
MDG	Madagascar	MDV	Maldives
MEX	Mexico	MKD	Macedonia
MLI	Mali	MLT	Malta
MNE	Montenegro	MNG	Mongolia
MOZ	Mozambique	MRT	Mauritania
MUS	Mauritius	MWI	Malawi
MYS	Malaysia	NAM	Namibia
NER	Niger	NGA	Nigeria
NIC	Nicaragua	NLD	Netherlands
NOR	Norway	NPL	Nepal
PAK	Pakistan	PAN	Panama
PER	Peru	PHL	Philippines
PNG	Papua New Guinea	POL	Poland
PRT	Portugal	PRY	Paraguay
ROU	Romania	RUS	Russian Federation
RWA	Rwanda	SDN	Sudan
SEN	Senegal	SLB	Solomon Isds
SLE	Sierra Leone	SLV	El Salvador
SRB	Serbia	STP	Sao Tome and Principe
SVK	Slovakia	SVN	Slovenia
SWE	Sweden	SWZ	Swaziland
SYR	Syria	TCD	Chad
TGO	Togo	THA	Thailand
TJK	Tajikistan	TLS	Timor-Leste
TON	Tonga	TUN	Tunisia
TUR	Turkey	TUV	Tuvalu
TZA	United Rep. of Tanzania	UGA	Uganda
UKR	Ukraine	URY	Uruguay
USA	USA	VEN	Venezuela
VNM	Viet Nam	VUT	Vanuatu
WSM	Samoa	YEM	Yemen
ZAF	So. African Customs Union	ZMB	Zambia
ZWE	Zimbabwe		

Table 2: Correlation matrix between explanatory variables

	Ln GDP per cap.	Gini index	Ln Pop.	Trade openness	Human capital	Aggregate TFP
Ln GDP per cap.	1					
Gini index	-.35	1				
Ln Pop.	-.02	.11	1			
Trade openness	-.02	-.16	-.20	1		
Human capital index	.79	-.45	-.1	.14	1	
Aggregate TFP	.81	-.33	.00	-.14	.49	1

Appendix C: A theoretical model of "vertical home market effect"

We model international trade between a domestic (D) and a foreign (F) country. Each country features a two-class society with N_r ($r = D, F$) consumers differing in their effective labor endowment, and hence belonging either to a poor (P) or a rich (R) class. The extent of inequality within each economy is determined by the share β_r of poor consumers within the population and by the distribution of the aggregate amount of effective labor supply L available in the economy.[14] $\theta_r \in (0, 1)$ is defined as the ratio of a poor consumer's labor supply l_{Pr} *relative* to the average per-capita labor supply L/N_r: $\theta_r = \frac{l_{Pr}}{L/N_r}$. As θ_r gets closer to 1, the level of inequality within the economy r diminishes. Given θ_r, it is possible to compute the labor supply of respectively a poor and a rich consumer in country r as $l_{Pr} = \theta_r \frac{L}{N_r}$ and $l_{Rr} = \frac{1-\beta_r\theta_r}{1-\beta_r} \frac{L}{N_r}$. In this framework, a mean-preserving increase in the level of inequality corresponds to a decrease in θ_r, while an increase in the average income, leaving the level of inequality unchanged, corresponds to a decrease in N_r.

The utility of a type i ($i = P, R$) consumer living in country r is assumed to be of the form:

$$U_{ir} = M_{ir}^{\mu} A_{ir}^{1-\mu} \tag{1}$$

with M_{ir} being an index of consumption of the varieties of a both vertically- and horizontally differentiated good, and A_{ir} being the consumed quantity of a homogenous good. The homogenous good is priced competitively, freely traded, and produced with unit efficient labor requirement, therefore implying that wages equalize across countries and can be normalized to 1. With $n = n_D + n_F$ being the total number of varieties of the differentiated good being produced (i.e. both domestic and foreign), we define M_{ir} as:

$$M_{ir} = \left[\int_0^n \left(\gamma_k^{\phi_i} c_{ir}(k) \right)^{\frac{\sigma-1}{\sigma}} dk \right]^{\frac{\sigma}{\sigma-1}}, \ \sigma \in (1, +\infty) \tag{2}$$

where γ_k and $c_{ir}(k)$ are respectively the quality and the quantity of a variety k consumed by a type i consumer living in country r, σ is the elasticity of substitution between any two varieties, and ϕ_i is a type-specific taste parameter that determines the intensity of preferences for product quality. Along Hallak (2006) and Lugovskyy and Skiba (2015), we assume that ϕ_i is a positive function of individual income l_i, i.e. that richer households value quality more.

Consumers use two-stage budgeting. A type i consumer living in country r will devote a share μ of its overall income l_{ir} to the consumption of the differentiated good; she will then spend the following amount of those expenses μl_{ir} on a given variety k:

$$p_r(k)c_{ir}(k) = \left(\frac{\left(\frac{p_r(k)}{\gamma_k^{\phi_i}} \right)^{1-\sigma}}{\int_0^n \left(\frac{p_r(k)}{\gamma_k^{\phi_i}} \right)^{1-\sigma} dk} \right) \mu l_{ir} \tag{3}$$

with $p_r(k)$ being the price charged for the variety k in country r. Assuming there exists only two possible qualities for each variety, i.e. high quality γ_H and low quality γ_L ($\gamma_H > \gamma_L$),

[14] Since we want to neutralize any supply-based variation in the quality mix being produced and exported, we assume that both countries have the same fixed amount of overall labor supply L.

using (3) and introducing specific consumption indices C_{ir}^L and C_{ir}^H for low- and high-quality variety bundles,[15] the share $s_j(l_{ir})$ of those expenses μl_{ir} devoted to varieties of quality j ($j = H, L$) is:

$$s_j(l_{ir}) = \frac{P_{rj} C_{ir}^j}{\mu l_{ir}} = \frac{\left(\frac{P_{rj}}{\gamma_j^{\phi_i}}\right)^{1-\sigma}}{\left(\frac{P_{rL}}{\gamma_L^{\phi_i}}\right)^{1-\sigma} + \left(\frac{P_{rH}}{\gamma_H^{\phi_i}}\right)^{1-\sigma}} \tag{4}$$

with $P_{rj} = \left[\int_0^{n_{rj}} p_{rj}(k)^{1-\sigma} dk + \int_0^{n_{sj}} p_{rj}^s(k)^{1-\sigma} dk\right]^{\frac{1}{1-\sigma}}$ ($r, s = D, F$, $r \neq s$) being a quality and country-specific price index, and p_{rj}^s being the price of a good of quality j produced in country s and sold in country r (p_{rj} being the mill price).

Focusing on the share devoted to high-quality goods, we have the following properties:

Proposition 1 (Properties of the expenditure share devoted to high-quality varieties in the case of a quality-augmented CES utility specification): *For a given set of prices (P_{rH}, P_{rL}), we have the following properties.*
Property 1 (P1): *The average propensity to consume high-quality varieties increases along income:* $\frac{\partial s_H(l_{ir})}{\partial l_{ir}} > 0$.
Property 2 (P2): *For levels of income l_{ir} for which we have $s_H(l_{ir}) < s_L(l_{ir})$, the share of expenditures devoted to the consumption of high-quality varieties is convex along income:* $\frac{\partial^2 s_H(l_i)}{\partial l_i^2} > 0$.

Proof. See Appendix D.

Hence, **high-quality varieties are goods whose share in a given consumer's consumption basket increases along individual income**. Moreover, **as long as the share of high-quality varieties is lower than the share of low-quality varieties in the consumption basket, the share of high-quality varieties increases along income in a convex way**.

Firms compete monopolistically. In the quality segment j, producing a quantity $x_j(k)$ of variety k requires $f_j + a_j x_j(k)$ units of labor, with f_j and a_j being respectively the fixed and marginal labor requirements for quality j.[16] We impose $a_H > a_L$, in line with the idea that high-quality varieties are more expensive to produce (see, for example Kugler and Verhoogen, 2012), and assume free entry in each segment of the market.[17] Our model features "iceberg" trade costs: in order to export to country r ($r \in \{D, F\}$) one unit of quality j's output manufactured in country s, a firm must ship $\tau_j \geq 1$ units. Finally, firms fully pass on their shipping costs to their foreign customers: one unit of variety k of quality j manufactured in country s is sold to consumers of country r at price $p_{sj}^r(k) = \tau p_{sj}$, where p_{sj} is the mill price. Denoting by $D_{rj} = \beta_r N_r C_{Pr}^j + (1-\beta) N_r C_{Rr}^j$ the total demand in country r for all varieties of quality j (both domestically- and foreign produced),

[15] $C_{ir}^j = \left(n_{Dj}(\gamma_j^{\phi_i} c_{ijr}^D)^{\frac{\sigma-1}{\sigma}} + n_{Fj}(\gamma_j^{\phi_i} c_{ijr}^F)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$ with c_{ijr}^r and n_{rj} denoting respectively the consumption for a variety of quality j ($j = H, L$) produced in country r by a type i consumer and the number of firms producing quality j within country r.

[16] Since we want to neutralize any supply-side determinant of a country's vertical specialization, we assume that those costs are similar across countries.

[17] We assume that firms are mono-variety in our set-up: a single firm cannot enter both quality segments of the market.

(3) yields the following expression for the demand d_{rj} devoted to a variety of quality j produced in country r: $d_{rj} = p_{rj}^{-\sigma}(P_{rj}^{\sigma}D_{rj} + \tau^{1-\sigma}P_{sj}^{\sigma}D_{sj})$. The resolution of the firm's profit maximization problem within each country and quality segment is similar to the benchmark monopolistic competition model, and yields the following standard mill price and break-even output:

$$p_{rj} = \frac{\sigma}{\sigma-1}a_j, \quad d_{rj} = \frac{f_j(\sigma-1)}{a_j} \tag{5}$$

The price index in country r for quality j can then be re-expressed as:

$$P_{rj} = (n_{rj} + \tau^{1-\sigma}n_{sj})^{\frac{1}{1-\sigma}}\frac{\sigma}{\sigma-1}a_j \tag{6}$$

It is then convenient to introduce along Fajgelbaum et al. (2011) the notion of "effective competitors" of quality j present on the domestic market r: $\tilde{n}_{rj} = n_{rj} + \tau^{1-\sigma}n_{sj}$. The intuition behind the concept is straightforward: while love for variety guarantees that for each quality j, each consumer in each country will devote a non-null part of its overall expenses to every available variety (both domestic and foreign), the market penetration of foreign varieties is discounted by a factor $\tau^{1-\sigma}$, capturing the fact that the price charged for foreign varieties bears the burden of shipping costs. Substituting for (6) in d_{rj}, we then get:

$$d_{rj} = \tilde{n}_{rj}^{\frac{\sigma}{1-\sigma}}D_{rj} + \tau^{1-\sigma}\tilde{n}_{sj}^{\frac{\sigma}{1-\sigma}}D_{sj} \tag{7}$$

Equating demand and supply within each country and quality segment and using the fact that $d_{Dj} = d_{Fj} = d_j$ (i.e. domestic demand faced by a producer of a quality j variety is the same in both countries), equations (4), (5) and (7) yield the following four equilibrium conditions:

$$\frac{f_j\sigma}{\mu L} = \frac{(1+\tau^{1-\sigma})(\beta_r\theta_r s_j(l_{Pr}) + (1-\beta_r\theta_r)s_j(l_{Rr}))}{\tilde{n}_{rj}}, \qquad j = H, L,\ r = D, F \tag{8}$$

Proposition 2 (Existence and uniqueness of the equilibrium with trade): *For given income distribution parameters β_r, N_r, L_r and θ_r ($r = D, F$), there exists a unique positive solution to the system of four equations defined by (8), determining the distribution of effective firms across countries and sectors ($\tilde{n}_{DL}, \tilde{n}_{DH}, \tilde{n}_{FL}, \tilde{n}_{FH}$).*

Proof. See Appendix C.

This result concerning the number of *effective* firms within each country does however not guarantee that we will observe trade of the differentiated good at the equilibrium. Indeed, we have the following expression for n_{rj}, i.e. the number of local firms producing varieties of quality j within country r:

$$n_{rj} = \frac{\tilde{n}_{rj} - \tau^{1-\sigma}\tilde{n}_{sj}}{1-\tau^{2(1-\sigma)}}, \qquad r \neq s,\ j = H, L,\ r, s = D, F \tag{9}$$

which entails the following condition for n_{rj} to be positive, i.e. to have partial specialization of both countries:

$$\tau^{1-\sigma} < \frac{\tilde{n}_{rj}}{\tilde{n}_{sj}} < \frac{1}{\tau^{1-\sigma}}, \qquad r \neq s,\ j = H, L,\ r, s = D, F \tag{10}$$

Condition (10) is scarcely respected for low levels of transport costs, i.e. τ very close to 1, but always met for high enough values of τ.[18] From now on, we hence assume the

[18] For low values of τ, condition (10) is respected when countries D and F are relatively similar in terms of average income $\frac{L_r}{N_r}$ and efficient labor size L_r.

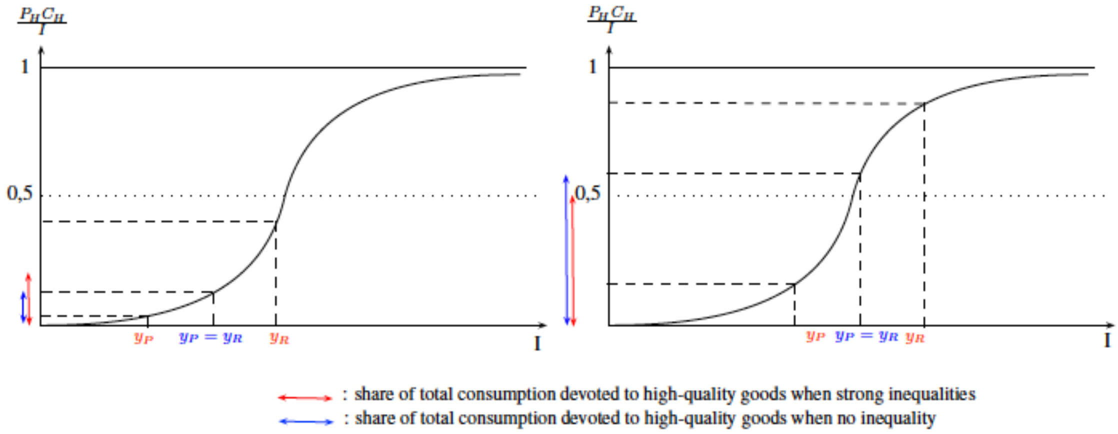


Figure 3: Heterogenous impact of inequality along the average income dimension

transport costs τ are sufficiently high to guarantee that both countries produce and export the two qualities, i.e. that $n_{rj} > 0$ for $j = H, L$ and $r = D, F$.

Proposition 3 (Impact of the average income and the level of inequality on the average quality of the export bundle):
For given income distribution parameters θ_D, N_D, N_F and θ_F and for high enough transport costs τ, we have the following comparative statics along N_D and θ_D:
(i) An increase in average income within country D (i.e. a decrease in N_D) generates an increase in the average quality of country D's export bundle: $\frac{\partial n_{DH}}{\partial N_D} < 0$, $\frac{\partial n_{DL}}{\partial N_D} > 0$.
(ii) Provided we have $s_H(l_{iD}) < s_L(l_{iD})$ for both $i = P, R$, a mean-preserving spread of income within country D (i.e. a decrease in θ_D) generates an increase in the average quality of country D's export bundle: $\frac{\partial n_{DH}}{\partial \theta_D} < 0$, $\frac{\partial n_{DL}}{\partial \theta_D} > 0$.

Proof. See Appendix D.

Proposition 3 implies that domestic income distribution has an impact on the quality mix being exported to trading partners. This result is the vertical translation of the classic horizontal "home-market effect" identified by Krugman (1980): a bigger market for varieties of a given quality j ensures the possibility to serve more consumers with sales that do not bear shipping costs, generating the entry of a greater number of producers of quality j and resulting in a shift in the quality level of exports.

Part (i) of Proposition 3 states that the average quality of the export bundle increases along the average income of consumers. This result is straightforward: since the share of overall consumption devoted to high-quality goods increases along income, an increase of average income leads to an increase in the size of the market for high-quality varieties. Such a demand shift raises the relative profitability of high-quality varieties, leaving the possibility for a higher number of firms to enter the market: $\tilde{n}_{DH}$ increases, leading to an increase (resp. decrease) in n_{DH} (resp. n_{DL}) and driving the exported quality mix upwards.

Part (ii) of Proposition 3 states that inequality has a positive impact on the exported quality mix provided both l_{PD} and l_{RD} are below the income threshold for which $s_H(l_{iD})$ becomes greater than $s_L(l_{iD})$, i.e. if the evolution of the income share devoted to high-quality varieties is convex along income for both rich *and* poor consumers (cf property (P2) of the consumers' preference system). This result is intuitively less straightforward, since mean-preserving variations in the spread of income impact in opposite ways the consumption of high-quality varieties of the poor and the rich: $\frac{\partial s_H(l_{PD})}{\partial \theta_D} > 0$, while $\frac{\partial s_H(l_{RD})}{\partial \theta_D} < 0$. As it can be seen from Figure 3, the properties of concavity/convexity of the evolution of a consumer's income share devoted to high-quality varieties following an increase in her income are then essential so as to grasp the mechanism at work. When the income share devoted to high-quality varieties increases in a convex way for *both* poor and rich consumers within the economy (cf graph on the left of Figure 3), the marginal increase of rich consumers' demand for high-quality varieties following an increase in inequality is more important than the marginal decrease of poor consumers' demand. Moreover, an increase in inequality gives more weight to rich consumers in total income. This leads to an overall increase in aggregate demand for high-quality varieties. As exemplified in Figure 3, the convexity of the Engel curve for high-quality varieties for both rich and poor consumers is more likely to be observed in poorer countrie. This property fails to follow through when the average income in the economy is high enough for $s_H(l_{RD})$ to increase in a concave way following a positive income shock (cf graph on the right of Figure 3).

As already mentioned in the main paper, those results regarding the effect of income and inequality on a country's quality mix are in line with those obtained by Fajgelbaum et al. (2011).[19] The nature of the adjustment of aggregate demand for high- and low- quality varieties is however different in the two models. In our model featuring love-for-variety at the individual level, it derives from changes in the quantity of each quality consumed at the individual level; in their model featuring heterogeneous consumers and unit consumption, it stems from changes in the number of people choosing a variety of a given quality.

Appendix D: Proofs of the theoretical model

Proof of Proposition 1

We have the following expressions for the different derivatives w.r.t. income l_i considered in Proposition 1:

$$\begin{aligned}
\frac{\partial s_H(l_i)}{\partial l_i} &= \frac{\partial \phi_i}{\partial l_i}\frac{\partial s_H(l_i)}{\partial \phi_i} = \frac{\partial \phi_i}{\partial l_i}(\sigma-1)s_H(l_i)s_L(l_i)(\ln(\gamma_H)-\ln(\gamma_L)) \\
\frac{\partial^2 s_H(l_i)}{\partial l_i^2} &= (\sigma-1)(\ln(\gamma_H)-\ln(\gamma_L))s_H(l_i)s_L(l_i)\left[\frac{\partial^2 \phi_i}{\partial l_i^2} + \left(\frac{\partial \phi_i}{\partial l_i}\right)^2(\sigma-1)(\ln(\gamma_H)-\ln(\gamma_L))(s_L(l_i)-s_H(l_i))\right]
\end{aligned}$$

Since $\gamma_H > \gamma_L$ and ϕ_i increases along l_i, we have unambiguously $\frac{\partial s_H(l_i)}{\partial l_i} > 0 \ \forall l_i > 0$. The sign of the second derivative depends on the sign of $s_L(l_i) - s_H(l_i)$ and $\frac{\partial^2 \phi_i}{\partial l_i^2}$: provided we have $s_H(l_i) < s_L(l_i)$ and the relationship between income and taste for quality is linear or convex, we hence have $\frac{\partial^2 s_H(l_i)}{\partial l_i^2} > 0$. This ends the proof. $\square$

[19] Indeed, the convexity property of the evolution of the income share devoted to high-quality varieties (needed so as to guarantee a positive impact of the inequality level on the average quality of the export bundle) is similar in the two models. In Fajgelbaum et al. (2011)'s unit consumption model, it implies that a majority of any income class purchases low-quality goods; in our model featuring love-for-variety at the individual level, this property is similarly verified for countries in which both rich and poor consumers devote a greater share of their income to low-quality varieties.

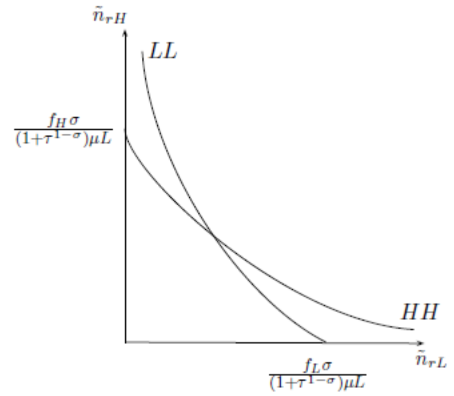


Figure 4: HH and LL in the $(\tilde{n}_{rH}, \tilde{n}_{rL})$ plane

0.1 Proof of Proposition 2

Using (4) and (5), it is possible to reformulate the share $s_j(l_{ir})$ devoted to the consumption of varieties of quality j of a type i consumer living in country r as:

$$s_j(l_{ir}) = \frac{a_j^{1-\sigma}\tilde{n}_{rj}\gamma_j^{\phi_i(\sigma-1)}}{a_H^{1-\sigma}\tilde{n}_{rH}\gamma_H^{\phi_i(\sigma-1)} + a_L^{1-\sigma}\tilde{n}_{rL}\gamma_L^{\phi_i(\sigma-1)}}$$

The equilibrium conditions featured in (8) represent the possible combinations for numbers of low- and high-quality effective producers consistent with market clearing and zero profits in the two market segments in both countries. More precisely, for a given country r we have:

$$\frac{f_L\sigma}{\mu L_r} = \frac{(1+\tau^{1-\sigma})(\beta_r\theta_r s_L(l_{Pr}) + (1-\beta_r\theta_r)s_L(l_{Rr}))}{\tilde{n}_{rL}} \tag{11}$$

$$\frac{f_H\sigma}{\mu L_r} = \frac{(1+\tau^{1-\sigma})(\beta_r\theta_r s_H(l_{Pr}) + (1-\beta_r\theta_r)s_H(l_{Rr}))}{\tilde{n}_{rH}} \tag{12}$$

(11) and (12) yield two implicit functions $\tilde{n}_{rH} = \psi^L(\tilde{n}_{rL})$ and $\tilde{n}_{rH} = \psi^H(\tilde{n}_{rL})$. ψ^L and ψ^H are implicitly defined by writing (11) and (12) as $L(\tilde{n}_{rH}, \tilde{n}_{rL}) = 0$ and $H(\tilde{n}_{rH}, \tilde{n}_{rL}) = 0$ with:

$$L(.) = -\frac{f_L\sigma}{(1+\tau^{1-\sigma})\mu L_r} + \frac{\beta_r\theta_r s_L(l_{Pr})}{\tilde{n}_{rL}} + \frac{(1-\beta\theta_r)s_L(l_{Rr})}{\tilde{n}_{rL}}$$

$$H(.) = -\frac{f_H\sigma}{(1+\tau^{1-\sigma})\mu L_r} + \frac{\beta_r\theta_r s_H(l_{Pr})}{\tilde{n}_{rH}} + \frac{(1-\beta\theta_r)s_H(l_{Rr})}{\tilde{n}_{rH}}$$

ψ_L and ψ_H can be represented as downward-sloping curves in the $(\tilde{n}_{rH}, \tilde{n}_{rL})$ plane (respectively LL and HH in figure 4), since an increase in the number of competitors in one quality segment necessarily leads to a decrease in the number of competitors in the other segment in order to preserve profitability. More precisely, we have $\tilde{n}_{rL} \to \frac{f_L\sigma}{(1+\tau^{1-\sigma})\mu L}$ as $\tilde{n}_{rH} \to 0$ and $\tilde{n}_{rL} \to 0$ as $\tilde{n}_{rH} \to \infty$ in (11), while we have $\tilde{n}_{rH} \to \frac{f_H\sigma}{(1+\tau^{1-\sigma})\mu L}$ as $\tilde{n}_{rL} \to 0$ and $\tilde{n}_{rH} \to 0$ as $\tilde{n}_{rL} \to \infty$ in (12). The two curves hence necessarily intersect in the positive quadrant, i.e. there exists a positive equilibrium with $\tilde{n}_{rH} > 0$ and $\tilde{n}_{rL} > 0$.

Such an equilibrium is unique if LL is always steeper than HH, i.e. if $\frac{\partial\psi_L}{\partial\tilde{n}_{rL}} < \frac{\partial\psi_H}{\partial\tilde{n}_{rL}}$ $\forall\tilde{n}_{rL} > 0$. Using the implicit function theorem, this amounts to showing that we have $\frac{\partial L}{\partial \tilde{n}_{rL}}\frac{\partial H}{\partial \tilde{n}_{rH}} - \frac{\partial H}{\partial \tilde{n}_{rL}}\frac{\partial L}{\partial \tilde{n}_{rH}} > 0$. We note that $\frac{\partial s_j(l_i)}{\partial \tilde{n}_{rj}} = \frac{1}{\tilde{n}_{rj}} s_j(l_i)s_{-j}(l_i)$, and use the following notations to simplify the demonstration:

$$\begin{aligned} E[s_j] &= \beta_r\theta_r s_j(l_{Pr}) + (1-\beta_r\theta_r)s_j(l_{Rr}) \\ E[s_H s_L] &= \beta_r\theta_r s_L(l_{Pr})s_H(l_{Pr}) + (1-\beta_r\theta_r)s_L(l_{Rr})s_H(l_{Rr}) \end{aligned}$$

We then have:

$$\begin{aligned} \frac{\partial L}{\partial \tilde{n}_{rL}}\frac{\partial H}{\partial \tilde{n}_{rH}} - \frac{\partial H}{\partial \tilde{n}_{rL}}\frac{\partial L}{\partial \tilde{n}_{rH}} &= \left(1/\tilde{n}_{rL}^2\right)\left(1/\tilde{n}_{rH}^2\right) E[s_L]E[s_H]\left(\left(\frac{E[s_H s_L]}{E[s_L]} - 1\right)\left(\frac{E[s_H s_L]}{E[s_H]} - 1\right) - \frac{E[s_H s_L]^2}{E[s_H]E[s_L]}\right) \\ &= \left(1/\tilde{n}_{rL}^2\right)\left(1/\tilde{n}_{rH}^2\right) E[s_L]E[s_H]\left(1 - \frac{E[s_L s_H]}{E[s_L]E[s_H]}\right) \end{aligned}$$

Using the fact that $s_L(l_{ir}) = 1 - s_H(l_{ir})$, we have $E[s_L]E[s_H] = E[s_H] - E[s_H]^2$, while $E[s_H s_L] = E[s_H] - E[s_H^2]$: we hence have $\frac{E[s_L s_H]}{E[s_L]E[s_H]} < 1$, and $\frac{\partial L}{\partial \tilde{n}_{rL}}\frac{\partial H}{\partial \tilde{n}_{rH}} - \frac{\partial H}{\partial \tilde{n}_{rL}}\frac{\partial L}{\partial \tilde{n}_{rH}} > 0$. This ends the proof. □

Proof of Proposition 3

Using the implicit function theorem, the comparative statics of $\tilde{n}_{DH}$ and $\tilde{n}_{DL}$ with respect to a parameter η ($\eta = N_D, \theta_D$) can be obtained with the formula:

$$\begin{pmatrix} \frac{\partial \tilde{n}_{DH}}{\partial \eta} \\ \\ \frac{\partial \tilde{n}_{DL}}{\partial \eta} \end{pmatrix} = -\begin{pmatrix} \frac{\partial H}{\partial \tilde{n}_{DH}} & \frac{\partial H}{\partial \tilde{n}_{DL}} \\ \\ \frac{\partial L}{\partial \tilde{n}_{DH}} & \frac{\partial L}{\partial \tilde{n}_{DL}} \end{pmatrix}^{-1} \begin{pmatrix} \frac{\partial H}{\partial \eta} \\ \\ \frac{\partial L}{\partial \eta} \end{pmatrix}$$

which yields:

$$\begin{pmatrix} \frac{\partial \tilde{n}_{DH}}{\partial \eta} \\ \\ \frac{\partial \tilde{n}_{DL}}{\partial \eta} \end{pmatrix} = -\frac{1}{\frac{\partial H}{\partial \tilde{n}_{DH}}\frac{\partial L}{\partial \tilde{n}_{DL}} - \frac{\partial H}{\partial \tilde{n}_{DL}}\frac{\partial L}{\partial \tilde{n}_{DH}}} \begin{pmatrix} \frac{\partial H}{\partial \eta}\frac{\partial L}{\partial \tilde{n}_{DL}} - \frac{\partial H}{\partial \tilde{n}_{DL}}\frac{\partial L}{\partial \eta} \\ \\ -\frac{\partial L}{\partial \tilde{n}_{DH}}\frac{\partial H}{\partial \eta} + \frac{\partial H}{\partial \tilde{n}_{DH}}\frac{\partial L}{\partial \eta} \end{pmatrix}$$

The sign of the fraction is straightforward: considering demonstration of proposition 1, we have $-\frac{1}{\frac{\partial H}{\partial \tilde{n}_{rH}}\frac{\partial L}{\partial \tilde{n}_{DL}} - \frac{\partial H}{\partial \tilde{n}_{DL}}\frac{\partial L}{\partial \tilde{n}_{DH}}} < 0$. We are left to determine the signs of the derivatives of H and L with respect to θ_D and N_D:

$$\begin{aligned} \frac{\partial L}{\partial N_D} &= \frac{\beta_D\theta_D}{\tilde{n}_{DL}}\left(\frac{\partial s_L}{\partial l_{PD}}\frac{\partial l_{PD}}{\partial N}\right) + \frac{(1-\beta_D\theta_D)}{\tilde{n}_{DL}}\left(\frac{\partial s_L}{\partial l_{RD}}\frac{\partial l_{RD}}{\partial N}\right) \\ \frac{\partial H}{\partial N_D} &= -\frac{\beta_D\theta_D}{\tilde{n}_{rH}}\left(\frac{\partial s_H}{\partial l_{PD}}\frac{\partial l_{PD}}{\partial N}\right) + \frac{(1-\beta_D\theta_D)}{\tilde{n}_{rH}}\left(\frac{\partial s_H}{\partial l_{RD}}\frac{\partial l_{RD}}{\partial N}\right) \\ \frac{\partial L}{\partial \theta_D} &= \frac{\beta_D}{\tilde{n}_{DL}}\left(s_L(l_{PD}) - s_L(l_{RD})\right) + \frac{\beta_D L_D}{N_D\tilde{n}_{DL}}\left[\theta_D\frac{\partial s_L}{\partial l_{PD}} - \frac{1-\beta_D\theta_D}{1-\beta_D}\frac{\partial s_L}{\partial l_{RD}}\right] \\ \frac{\partial H}{\partial \theta_D} &= \frac{\beta_D}{\tilde{n}_{rH}}\left(s_H(l_{PD}) - s_H(l_{RD})\right) + \frac{\beta_D L_D}{N_D\tilde{n}_{rH}}\left[\theta_D\frac{\partial s_H}{\partial l_{PD}} - \frac{1-\beta_D\theta_D}{1-\beta_D}\frac{\partial s_H}{\partial l_{RD}}\right] \end{aligned}$$

(i) We have $\frac{\partial l_{PD}}{\partial N_D} = -\theta_D\frac{L}{N_D^2} < 0$ and $\frac{\partial l_{RD}}{\partial N_D} = \frac{1-\beta_D\theta_D}{1-\beta_D}\frac{L}{N_D^2} < 0$. Along P1, we are further able to state that $\frac{\partial s_H(l_{iD})}{\partial l_{iD}} > 0$ and $\frac{\partial s_L(l_{iD})}{\partial l_{iD}} < 0$. We hence obtain unambiguously that $\frac{\partial L}{\partial N_D} > 0$ and $\frac{\partial H}{\partial N_D} < 0$. The implicit function theorem then entails that $\frac{\partial \tilde{n}_{rH}}{\partial N_D} < 0$ and $\frac{\partial \tilde{n}_{DL}}{\partial N_D} > 0$.

An alternative and more intuitive demonstration of part (i) of Proposition 2 can be obtained by considering a slightly modified version of the equilibrium condition (12):

$$\frac{f_H \sigma \tilde{n}_{DH}}{\mu L_D (1+\tau^{1-\sigma})} = \beta_D \theta_D s_H(l_{PD}) + (1-\beta_D \theta_D) s_H(l_{RD}) \tag{13}$$

As already said, an increase in N_D decreases both l_{PD} and l_{RD}, and hence generates a decrease of both $s_H(l_{PD})$ and $s_H(l_{RD})$ (cf property P1). The RHS of condition (13) hence unambiguously decreases. Considering the concavity of $s_H(l_{iD})$ along $\tilde{n}_{DH}$ ($\frac{\partial^2 s_H(l_{iD})}{\partial \tilde{n}_{DH}^2} < 0$, cf demonstration of Proposition 1) and the fact that the LHS is linear in $\tilde{n}_{DH}$, such a decrease of the RHS cannot be compensated by an increase in $\tilde{n}_{DH}$. The LHS necessarily needs to decrease for the equality to be respected again, leading to a decrease in $\tilde{n}_{DH}$ following an increase in N_D.

(ii) As stated in Proposition 2, we place ourselves in the case where both l_{RD} and l_{PD} are under the income threshold l^T beyond which we have $s_H(l^T) > s_L(l^T)$. Along P1 and since $l_{RD} > l_{PD}$, we have that $s_H(l_{PD}) - s_H(l_{RD}) < 0$ and $s_L(l_{PD}) - s_L(l_{RD}) > 0$. Along (P2), we have that $\frac{\partial s_H}{\partial l_{RD}} > \frac{\partial s_H}{\partial l_{PD}}$ and $\frac{\partial s_L}{\partial l_{RD}} < \frac{\partial s_L}{\partial l_{PD}}$. Using those properties, we can deduce $\frac{\partial L}{\partial \theta_D} > 0$ and $\frac{\partial H}{\partial \theta_D} < 0$. Considering the formula obtained with the implicit function theorem, we then obtain unambiguously that $\frac{\partial \tilde{n}_{DH}}{\partial \theta_D} < 0$ and $\frac{\partial \tilde{n}_{DL}}{\partial \theta_D} > 0$.

Adding up the equilibrium conditions in both quality segments for country D yields the following condition that needs to be met at the equilibrium:

$$f_L \sigma \tilde{n}_{DL} + f_H \sigma \tilde{n}_{DH} = \mu L_D (1+\tau^{1-\sigma}) \tag{14}$$

Hence, at fixed overall labor supply L_D, condition (14) guarantees that an increase in $\tilde{n}_{DH}$ is only possible through a decrease in $\tilde{n}_{DL}$. Furthermore, we have that:

$$\frac{\partial n_{rj}}{\partial \tilde{n}_{rj}} > 0 \qquad j = H, L,\ r = D, F \tag{15}$$

Those comparative statics imply that, provided that we are in an equilibrium with partial trade specialization (i.e. for high enough values of τ), an increase in the number $\tilde{n}_{rj}$ of "effective" producers of a given quality j in country r increases the number n_{rj} of domestic producers of this quality. We can hence directly interpret an increase in $\tilde{n}_{DH}$ as an increase in n_{DH}, and a decrease in $\tilde{n}_{DL}$ as a decrease in n_{DL}. In other words, an increase in $\tilde{n}_{DH}$ leads to a shift of the export mix of the domestic country D towards high a higher average quality at the equilibrium. This ends the proof. □